Introduction to Computational and Algorithmic Thinking

CHAPTER 8 – DATA REPRESENTATION AND COMPRESSION

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Announcements

This lecture: Data Representation and Compression

Reading: Read Chapter 8 of Conery

Acknowledgement: Some of this lecture slides are based on CSE 101 lecture notes by Prof. Kevin McDonald at SBU

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Data and Computers

- Computers are multimedia devices, dealing with a vast array of information categories
  - **Information** is data (basic values, facts) that has been organized or processed into useful form
- Computers store, present and help us modify various kinds of data: numbers, text, audio, images and graphics, video
- Information can be represented in one of two ways: analog or digital
- **Analog** data: a continuous representation, *analogous* to the actual information it represents
- **Digital** data: a discrete representation that breaks the information up into separate elements

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Analog vs. Digital

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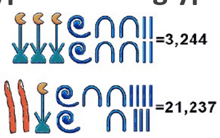
Representing Numbers

Human beings have contrived a wide variety of ways to represent the digits that make up numbers

Egyptian hieroglyphs:



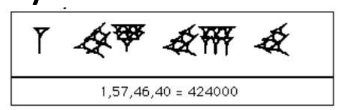
Roman numerals:

MMXVII = 2017

References:

- http://goo.gl/BSrWTH
- http://goo.gl/r8NcKF
- Wikipedia

Babylonian numerals:



Many others!

European	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
Tamil		௧	௨	௩	௪	௫	௬	௭	௮	௯

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Positional Notation

- The modern Western style and some other styles of writing numbers use **positional notation**
  - The position of a digit determines how much it contributes to the number's value
- With **decimal** (base 10), **place-values** are powers of 10:

  …, 10^3, 10^2, 10^1, 10^0, 10^{-1} , 10^{-2}, 10^{-3}, …

  …, 1000s, 100, 10s, 1/10 s, 1/100 s, 1/1000 s, …
- 642.15 really means $(6 \times 10^2) + (4 \times 10^1) + (2 \times 10^0) + (1 \times 10^{-1}) + (5 \times 10^{-2})$
- Early computers represented numbers with base 10, but they were very unreliable. It was too hard to make the computer maintain 10 distinct voltages for the 10 digits.

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Binary Numbers

- Modern digital computers use **binary digits** (base-2 numbers: 0 and 1)
  - The word **bit** is short for **binary digit**
- The hardware determines how bits are stored
  - Hard drive: magnetized spots on surface of disk
  - Flash drive: presence/absence of electrons in a memory cell
  - Optical disc (CD/DVD): pits and lands (flat spots)
- As computational thinkers, we do not need to worry so much about *how* the bits are stored
- Instead, we will concern ourselves about *what* bits are stored

Binary Numbers

With binary we have just two digits, 0 and 1, and the place-values are powers of 2:

…, 2^3, 2^2, 2^1, 2^0, 2^{-1}, 2^{-2}, 2^{-3}, …

…, 8s, 4s, 2s, 1s, 1/2s, 1/4s, 1/8s, …

The number 1011.011_2 written in decimal is:

$(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= 8 + 0 + 2 + 1 + 0 + 1/4 + 1/8 = 11.375_{10}$

Important observation: 1011.011_2 and 11.375_{10} are two different representations of the same quantity

All data in a modern machine is stored using binary numbers

Decimal → Binary Conversion

- To convert a decimal number to binary, perform these steps:
  1. Repeatedly divide the decimal number by 2.
  2. Set aside the remainder of each division.
  3. Use the quotient for the next round of division.
  4. When the quotient reaches 0, write down all of the remainders in order from last to first. This value is your answer.
- This algorithm is most easily understood by seeing some examples.

Decimal → Binary Example #1

Convert 123_{10} to binary

123 / 2 = 61 rem. 1

61 / 2 = 30 rem. 1

30 / 2 = 15 rem. 0

15 / 2 = 7 rem. 1

7 / 2 = 3 rem. 1

3 / 2 = 1 rem. 1

1 / 2 = 0 rem. 1

Answer: 1111011_2

Note that we write the remainders in the reverse order of how they are generated

The Function dec2bin()

Let's see a function **dec2bin()** that returns a string of 0s and 1s giving the binary representation of an integer

```
def dec2bin(decimal):
    binary = ""
    while decimal > 0:
        remainder = decimal % 2
        binary = str(remainder) + binary
        decimal = decimal // 2
    return binary

print(dec2bin(23)) # "10111"
print(dec2bin(100)) # "1100100"
```

data_rep.py

Encoding Data (next)

- To store information in a computer's memory we have to encode it somehow: an **encoding** is a pattern of 0s and 1s
  - The pattern is a representation of some real-world object, like a letter, number, sound clip, or video
- Encoding is not the same as **encryption**
  - Both use codes, but in this slide set we will explore standard ways of *representing* data, not *hiding* data
- A set of *k* bits can represent up to 2^k items. Let's see why.

  Each bit can be 0 or 1 (two options)

  With 2 bits, we can represent $2^2 = 4$ items

  With 3 bits, we can represent $2^3 = 8$ items

  …
- With k bits, we can represent 2^k items

Representing Characters

- **ASCII** (American Standard Code for Information Interchange) includes 7-bit and 8-bit schemes for representing characters used in the English language
- Each letter, number, punctuation mark, etc. is mapped to a 7-bit number
  - See ASCII.txt
- Examples: capital letter “A” is 65; lowercase letter “a” is 97
- A newer scheme called **Unicode** includes codes for 135 alphabets
  - Modern languages (Greek, Cyrillic, Arabic, Hebrew, Korean, Chinese, Japanese, ...) and ancient languages (hieroglyphics, runes, ...)
  - Also include technical symbols, emoji, and other symbols

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Representing Characters

- Here are three ways to include a Unicode symbol in a Python string:

1. Copy and paste text from an e-mail, a web page, etc.
2. Use a function named **chr** (short for “character”)
   - Pass it a code number. It will return a one-letter string containing that symbol. Example: **chr(9829)** gives '♥'
   - Find code numbers at www.charbase.com or similar websites that have lists of Unicode symbols
3. Use an escape sequence **‘\uXXXX’** where **XXXX** is the 4-digit **hexadecimal** (base 16) code number

- **'I \u2665 cats' is 'I cats’**
- See data_rep.py for ♥ more examples

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Hexadecimal Numbers

- In some software development it's more natural to write numbers in base 16, called **hexadecimal**
- With hexadecimal we have 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and the letters A through F for ten through fifteen
- Place-values in hexadecimal are powers of 16:
  …, 16^3, 16^2, 16^1, 16^0, 16^{-1}, 16^{-2}, 16^{-3}, …
- $51E_{16} = (5 \times 16^2) + (1 \times 16^1) + (14 \times 16^0) = 1{,}310_{10}$
- $FAD_{16} = (15 \times 16^2) + (10 \times 16^1) + (13 \times 16^0) = 4{,}013_{10}$
- Changing the base of a number doesn't change the magnitude (value) of a number
- The representation for a number gets longer as the base decreases.

Same Value, Different Base

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

Base 10	Base 2	Base 16
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Hexadecimal Numbers

- Hexadecimal is used widely in web design for giving colors
  - You can use it with Python too, as we just saw
- When giving the Unicode for a character as an escape sequence with **\u**, we always use hexadecimal
- In contrast, the **chr()** function expects the decimal representation
- For the heart symbol above, we used **9829** (base 10) for **chr()**, but **2665** (base 16) for the escape sequence
- The related **ord()** function returns the Unicode value of a character in decimal format:
  - **ord('A')** returns **65** and **ord('x')** returns **120**
- See data_rep.py for more examples

Hexadecimal Numbers

- The binary representation for a string can be hard to read:
- **01001001 00100111 01101101 00100000 01100001 01100110 01110010 01100001 01101001 01100100 00100000 01101111 01100110 00100000 01100011 01101111 01110111 01110011 00101110**
- It's a little easier to deal with codes in hexadecimal:

  49 27 6D 20 61 66 72 61 69 64 20 6F 66 20 63 6F 77 73 2E
- Recall that in hexadecimal we have 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and the letters A through F for ten through fifteen
- Note that each hexadecimal digit corresponds with four binary digits (bits)

Binary ↔ Hexadecimal

Base 2	Base 16
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

- To convert a hexadecimal number to a binary number, simply convert each digit in the hexadecimal number into a four-digit binary number.
- For example:

  $\mathbf{D2B5}_{16} = \mathbf{1101001010110101}_2$
- To convert a binary number to a hexadecimal, convert every four binary digits from **right to left** in the binary number into a hexadecimal digit.
- For example: $\mathbf{010001111110}_2 = \mathbf{47E}_{16}$

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Groups of Bits

- A **byte** is a collection of 8 bits
  - A 7-bit ASCII value fits in single byte
- A 32-bit integer requires 4 bytes (32 ÷ 8 = 4)
- A central processing unit (CPU) operates on several bytes at a time, called a **word**
  - A **word** is a collection of two or more bytes
  - Typical word size are 32 bits (4 bytes) and 64 bits (8 bytes)
- Memory capacity is often described in terms of *megabytes* or *gigabytes*

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Error Detection

- Errors in values can be caused by circumstances beyond our control
  - The storage medium itself has a flaw or is deteriorating
  - Data can be corrupted by interference during transfer over wires or wirelessly
  - Even solar activity itself can affect electronic devices and disrupt electronic communication
- The general method for detecting errors is to add extra information to the data
  - Add extra data to a document before storing it in a file
  - Append error checking data to a message while sending it

Error Detection

- The basic procedure for enabling error-free communication:
  1. Sender adds error-checking information
  2. After receiving the message, the receiver analyzes the message along with the extra data to see if an error occurred
  3. If an error occurred, the receiver will ask the sender to send the message again
- A simple method for error-checking is to use a **parity bit**
  - Add one extra bit to the end of the text
  - Here, “text” means any string: an entire message or a single character

Error Detection

- The value of the extra bit should make the total number of '1' bits an even number
  - This property is called **even parity**
- Example: parity bits for 8-bit ASCII characters
  - A = 01000001. There are two 1 bits, so attach a 0 as the parity bit (the total # of 1s remains two)
  - C = 01000011. There are three 1 bits, so attach a 1 as the parity bit (bringing the total # of 1s to four)
- Example: parity bit for a piece of DNA (using ASCII)
  - ATG = 01000001 01010100 01000111 + 1
  - The binary code for the entire string contains 9 1's, so we add one more 1 bit to make an even number of 1s

Error Detection

- The receiver treats the parity bit like any other bit in the incoming message
  - It is included in the count of the number of 1 bits
  - To get the message contents, the receiver discards the last bit
- Example: when sending an 8-bit ASCII 'C', the bit stream is 010000111: the digits in the code for 'C' plus a parity bit
  - The receiver reads 9 bits and sees there was an even number of 1 bits; no error detected
  - The receiver discards the 9th bit
  - The remaining bits are the contents of the message: 01000011, which is the ASCII code for 'C'
- **Note:** It is a very simple scheme. It can only be used to detect single or any other odd number (i.e., three, five, etc.) of errors in the output. An even number of flipped bits (errors) will make the parity bit appear correct even though the data is erroneous.

Aside: Communication Protocols

- For this error-checking plan to work, the sender and receiver both have to agree on a **communication protocol**
  - The protocol defines a message structure and also specifies what actions are taken during the transmission or receipt of a message
  - In our simple protocol, the sender and receiver agree in advance the parity bit is the last bit
- Two of the most important protocols used today are **Transmission Control Protocol (TCP)** and **Internet Protocol (IP)**
  - Used extensively in Internet communication, including the Web and online video games

Computing Parity Bits

- How can we write a function that computes the parity bit for a message?
- Deciding whether to attach a 1 or a 0 to the end of a code is simple using a logic function called "**exclusive or**", abbreviated as **XOR**
  - Normally, **a or b** is true if either **a** or **b** is true, or if both of **a** and **b** are true
  - The XOR of variables **a** and **b** is true if either one of them is true, but not if both are true
- XOR in Python is denoted using the caret, **^**

Computing Parity Bits

- It is uncommon in programming to use XOR in Boolean expressions (True/False expressions)
- Rather, XOR is used (almost) exclusively in bitwise operations, which are expressions that involve 0s and 1s
- In Python, bitwise-**and** is denoted by the ampersand, **&**, and bitwise **or** is denoted by the vertical bar, or pipe, **|**

	AND	OR	XOR
a b	a & b	a | b	a ^ b
0 0	0	0	0
0 1	0	1	1
1 0	0	1	1
1 1	1	1	0

Computing Parity Bits

- Let's consider a function that computes a parity bit. Here's the algorithm:

1. Initialize the return value **p** to 0.
2. Iterate over all the bits in the input code, updating **p** using the XOR operator: **p = p ^ bit**
   a. If a bit is 0, it won't change **p**.
   b. But if it's a 1, it sets **p** to the opposite value.

- Here's why this works. We start with **p = 0**. Every time we see a 1, we "flip" the parity bit by replacing **p** with
  p XOR 1.
- For example, if the data contain three 1s, then **p** will flip three times: **p** = 0 ➔1 ➔ 0 ➔ 1, so the parity bit will be 1

The parity() Function

- Given a string containing only 0s and 1s, the **parity()** function computes the parity bit

```
def parity(bits):
    p = 0
    for bit in bits :
        p = p ^ int(bit)
    return p
```

- Examples:
- **parity('1000001') # returns 0**
- **parity('1000011') # returns 1**
- See data_rep.py

Groups of Bits

- We can define our own encoding schemes for objects
- Suppose we wanted to encode DNA sequences, which are strings containing the letters A, C, G, and T
- We need only 2 bits to represent 4 different things
- We could create a dictionary to map a letter to a 2-bit code
- Example: "A" → 00
  "C" → 01
  "G" → 10
  "T" → 11
- We will now look at a famous algorithm for compressing data which produces a new, original binary encoding scheme for a given input data-set

Text Compression

- Data compression algorithms reduce in the amount of space needed to store a piece of data
- A data compression technique can be:
  - **Lossless** (no information lost)
  - **Lossy** (information lost)
- There are many algorithms for compressing files (including photos, images and other types of data) but we'll focus on a lossless technique for text compression called **Huffman coding**
- We will first need to explore a few data structures before we can understand how Huffman coding works

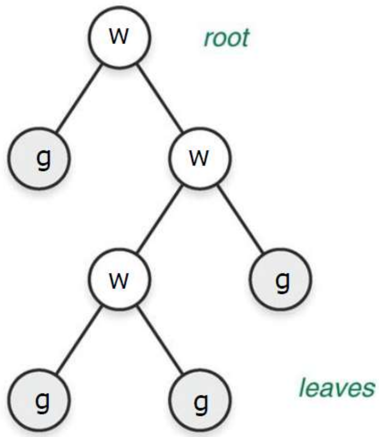
Binary Trees (next)

- In mathematics and computer science, a **tree** consists of data values stored at **nodes**, which are connected to each other in a hierarchical manner by **edges**
- Like a family tree, a tree shows parent-child relationships
- Each node in the tree, except for a special node called the **root**, has exactly *one* **parent node**
- Nodes can be connected to 0 or more **child** nodes immediately beneath them in the tree
- A node with at least one child is called an **interior node** (colored 'w'hite in the figure on the next slide)
- Towards the bottom of a tree we find nodes with no children; such nodes are called **leaves** (shaded 'g'ray in the figure on the next slide)

Binary Trees

- In a **binary tree**, every node has either 0, 1 or 2 children

- Used in this context, the word "binary" refers to the maximum number of children that a node can have. It does not refer to bits.



Huffman Coding

- Huffman coding is a scheme for encoding letters based on the idea of using shorter codes for more commonly used letters
  - ASCII uses 7 or 8 bits to store every letter, regardless of how often that letter is used in real text
  - Imagine if we could find a way to store commonly used letters like R, S, T, N, L, E, etc. using fewer bits
- For large data-sets consisting only of characters, the potential savings is huge
  - This is what Huffman coding accomplishes

Huffman Coding

- A **Huffman tree** is a binary tree that is at the heart of Huffman coding
- Inside of each node of a Huffman tree we store (i) a letter and (ii) the frequency of how often that letter appears in words

The Hawaiian Alphabet

- We will use the Hawaiian alphabet as part of a running example to understand how Huffman coding works
- Hawaiian words are spelled with the five vowels A, E, I, O, and U, and only the seven consonants H, K, L, M, N, P, and W
- The ' symbol, called the *okina*, is used between two vowels when they should be pronounced as separate syllables
  - Example: “a’a” is pronounced “ah-ah”

The Hawaiian Alphabet

- The table to the right shows the frequency of each letter in Hawaiian words
- We will exploit this knowledge to find an efficient encoding of the 13 symbols

Letter	Frequency
'	0.068
A	0.262
E	0.072
H	0.045
I	0.084
K	0.105
L	0.044
M	0.032
N	0.083
O	0.107
P	0.030
U	0.059
W	0.009

Data Structures for Huffman Coding

- In an earlier lecture we learned about a special kind of list called a *priority queue*
- Every item inserted into a priority queue has a corresponding numerical priority
- The priority queue always makes sure that the item with highest priority is at the front of the list
- The **PriorityQueue** class in the SpamLab implements the priority queue concept
- The **insert()** method adds an item to the priority queue
- The **pop()** method removes the item at the front of the list, which is guaranteed to be the item of highest priority

Data Structures for Huffman Coding

- We will use a priority queue to help us build a Huffman tree
- In the BitLab lab there is a class called **Node** we can use to build Huffman trees
  - When creating a **Node** object, we give the letter and the letter's frequency, as in this example:

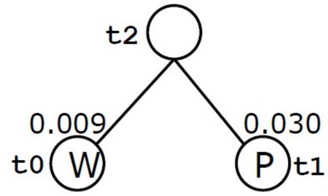
```
from PythonLabs.BitLab import Node
leaf = Node('M', 0.032)
```

- The above **Node** object creates a leaf node
- The Huffman coding algorithm will take a set of such nodes, one per letter, and insert them into a priority queue

Data Structures for Huffman Coding

- The priority queue will put the node with ***lowest*** frequency at the front of the list
- In other words, a letter's frequency will serve as its "priority", with high-frequency letters having the lowest priority
- If we want to create an interior node, which has one or two children, we have to "tell" the **Node** object which nodes are its children, as in this example:

```
t0 = Node('W', 0.009)
t1 = Node('P', 0.030)
t2 = Node(t0, t1)
```



Huffman Coding: The Algorithm

1. Make leaf nodes for every symbol in the alphabet
2. Put these nodes into a priority queue
3. Remove the first two nodes from the queue
4. Create a new interior node using these two nodes
5. Insert the new node back into the queue.
   - If there are still two more nodes in the queue, go to step 3
   - Otherwise, stop

- Let's see how this would work if we consider only the vowels (to make the example simpler)

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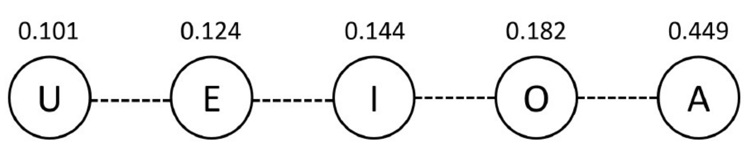
Huffman Coding: Example #1

- Below is the priority queue that would be created, with the front of the queue on the left:
- We see that U and E are the two front nodes



- So, we remove them from the queue, create a new interior node, and insert the new node into the queue, as we'll see on the next slide

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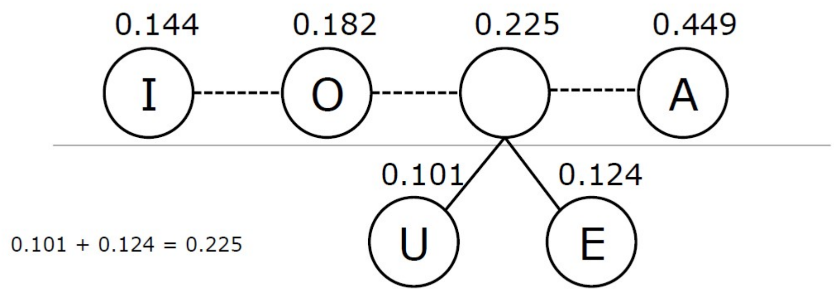
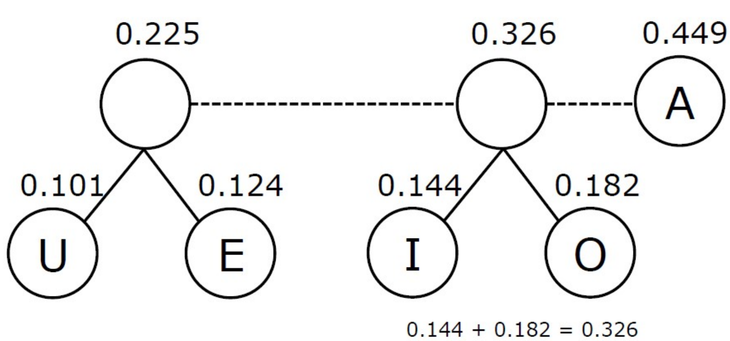
Huffman Coding: Example #1



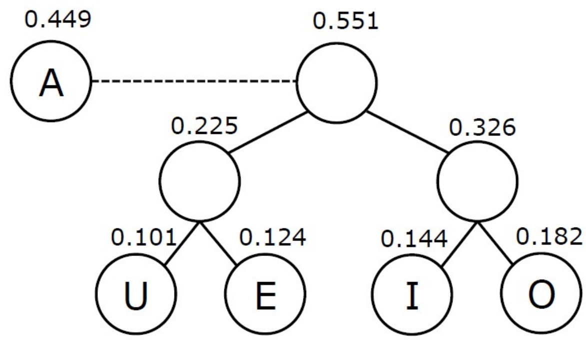
- Above the horizontal line is the content of the priority queue
- Note how the queue has one fewer entry in it now
- Next we'll remove the nodes for I and O, create a new node with these two nodes as children and add the new node back into the queue

Huffman Coding: Example #1



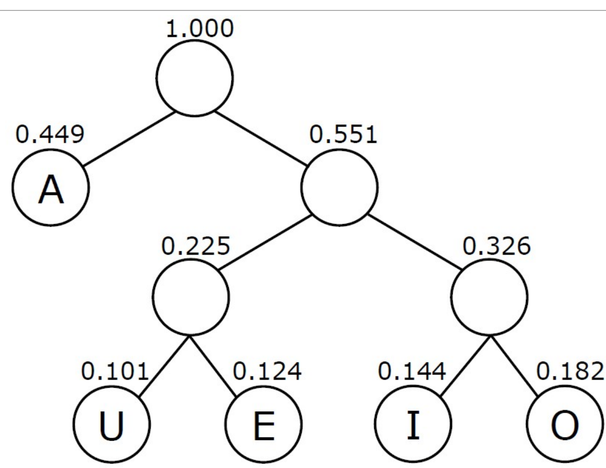
- Next we'll remove the nodes with the weights 0.225 and 0.326, and combine them into a new interior node

Huffman Coding: Example #1



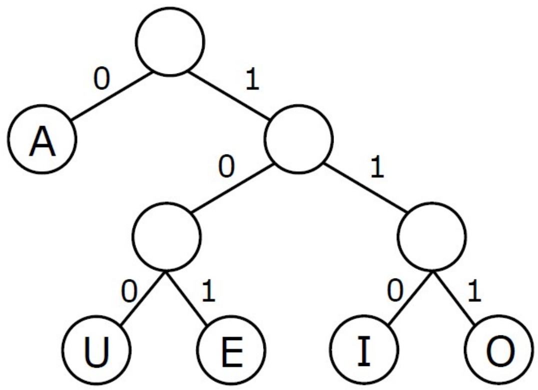
- Finally, we have only two nodes left, so we remove them both, and combine them into a new interior node
- This last node we create becomes the root of the binary tree

Huffman Coding: Example #1



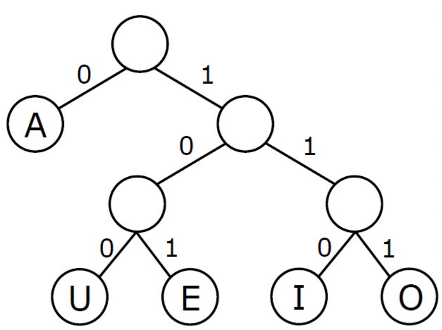
Huffman Coding: Example #1

- With the tree completed, we now attach 0's and 1's to the edges connected to the **left child** and **right child** of each node, respectively



Huffman Coding: Example #1

- Starting at the root, we trace the **path** from the root to each node to generate the codes for each letter:



Letter	Huffman Code
A	0
E	101
I	110
O	111
U	100

The build_tree() Function

- We can now implement a function **build_tree()** that will build a Huffman tree from the list of frequencies
- The function **read_frequencies()** from the BitLab module will load the frequencies stored from a file into a dictionary
- The **build_tree()** function then adds the frequencies into **Node** objects, which are in turn added into the priority queue
- Finally, a while-loop assembles the Huffman tree by removing items two at a time from the priority queue and re-inserts the resulting "merged" pairs back into the queue

The build_tree() Function

```
from PythonLabs.BitLab import Node, read_frequencies, init_queue
def build_tree(filename):
  pq = init_queue(read_frequencies(filename))
  while len(pq) > 1:
    n1 = pq.pop() # remove 1st element
    n2 = pq.pop() # remove 2nd element
    pq.insert(Node(n1,n2))
  return pq[0]
```

See huffman.py

Huffman Coding: Example #1

- Let’s try the function with the vowel frequencies:

```
vt = build_tree('hvfreq.txt')
print(vt)
# hvfreq.txt is in PythonLabs/data/huffman/
```

- Output:

```
( 1.000 ( A: 0.449 ) ( 0.551 ( 0.225
( U: 0.101 ) ( E: 0.124 ) ) ( 0.326 ( I: 0.144 )
( O: 0.182 ) ) ) )
```

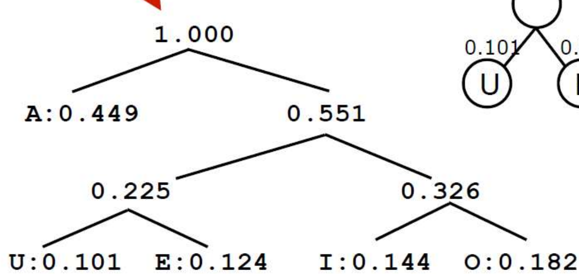
- Although it may not seem like it, this is actually our tree
- Let’s reformat it a little (see next slide)

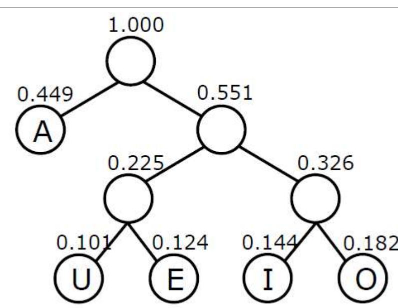
hvfreq.txt

```
A  0.449
E  0.124
I  0.144
O  0.182
U  0.101
```


Huffman Coding: Example #1

Program output reformatted. Look familiar?





Huffman Coding: Example #1

- Finally, the *recursive* function **assign_codes()** from BitLab assembled the Huffman codes from the Huffman tree:

```
from PythonLabs.BitLab import assign_codes
codes = assign_codes(vt)
print(codes)
```

- Output:

{'A': 0, 'E': 101, 'I': 110, 'O': 111, 'U': 100}

Huffman Coding: Example #2

- The file hafreq.txt contains the frequencies for all letters in the Hawaiian alphabet
- Let's build the Huffman tree from the frequencies:

```
at = build_tree('hafreq.txt')
```

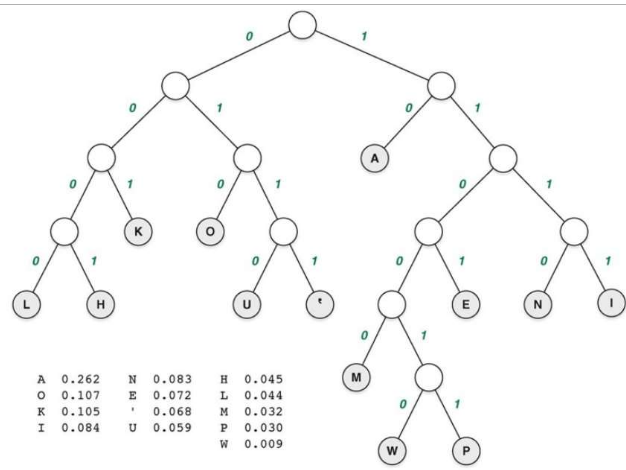
- Then assign the codes:

```
codes = assign_codes(at)
```

- Result: **{**

"'": 0111, 'A': 10, 'E': 1101,
'H': 0001, 'I': 1111, 'K': 001,
'L': 0000, 'M': 11000, 'N': 1110,
'O': 010, 'P': 110011, 'U': 0110,
'W': 110010 }

Huffman Coding: Example #2



A	0.262	N	0.083	H	0.045
O	0.107	E	0.072	L	0.044
K	0.105	'	0.068	M	0.032
I	0.084	U	0.059	P	0.030
				W	0.009

Huffman Coding: Example #2

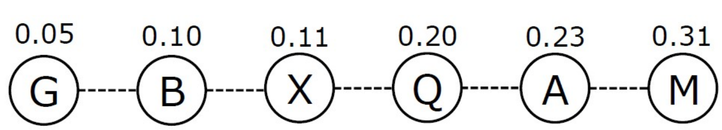
- What we find is that the most-frequently appearing letters have short codes, while the less-frequently appearing letters have longer code
- Also of note: no code is the prefix of another code
- For example, the code for A is 10. No other code begins with 10.
  - This fact is important when we want to decode a message
  - Let’s see now how we decode a message (next slide)

Huffman Coding: Example #2

- Suppose we have the message 110001001101111
- We scan the digits from left to right
- The first five digits, 11000, form the code for “M”
- The next two digits, 10, form the code for “A”
- The next four digits, 0110, form the code for “U”
- Finally, the last four digits, 1111, form the code for “I”
- So, the original encoded word was “MAUI”
- There is no other way to decode that string of bits to generate a different word

Huffman Coding: Example #3

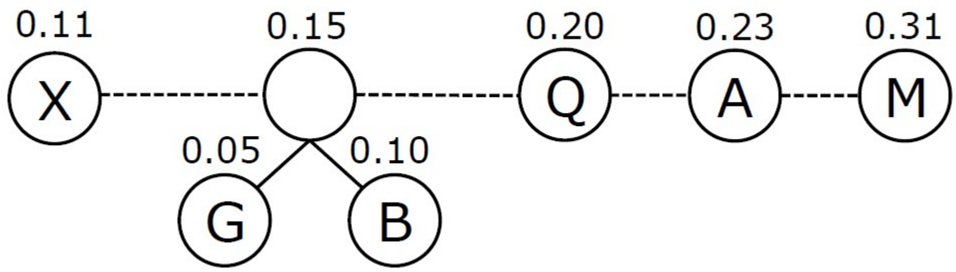
- Given the following letter frequencies, let’s compute the Huffman coding for the letters
- We begin by inserting the letters into a priority queue:



Letter	Frequency
A	0.23
B	0.10
G	0.05
M	0.31
Q	0.20
X	0.11

- Now merge the 2irst two elements in the queue until the tree is assembled (see few next slides)

Huffman Coding: Example #3



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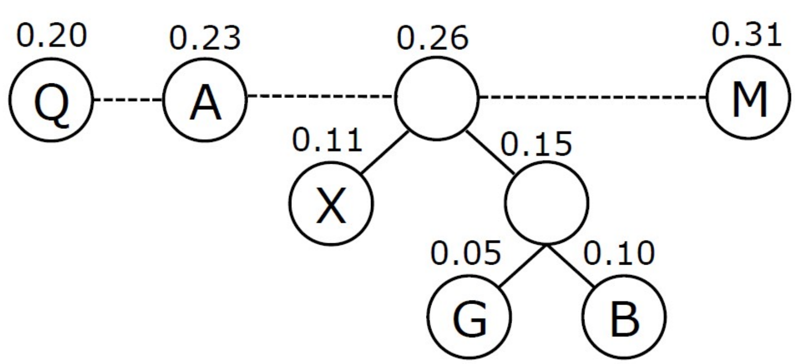
Huffman Coding: Example #3



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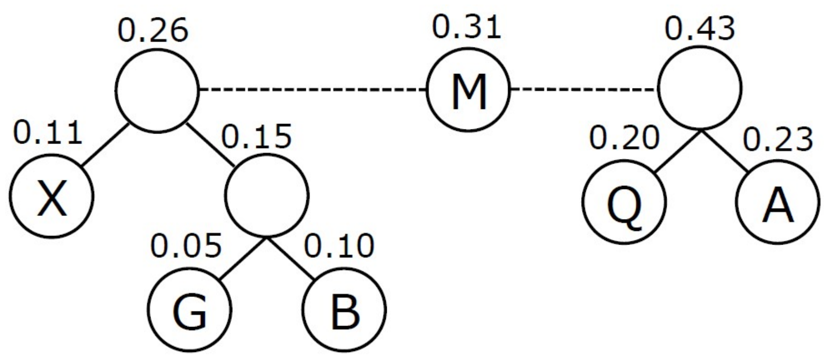
Huffman Coding: Example #3



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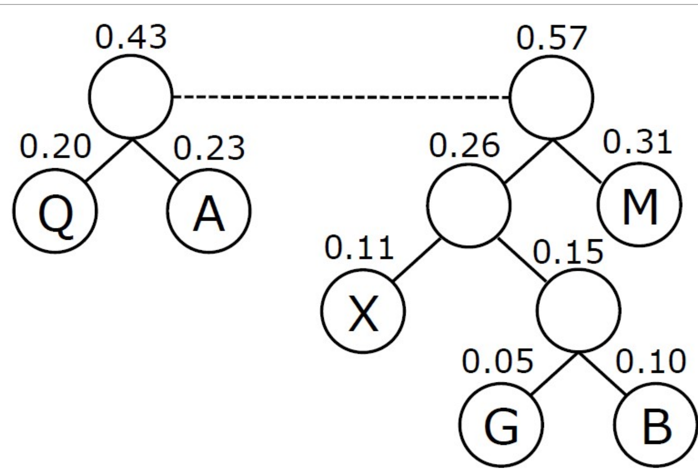
Huffman Coding: Example #3



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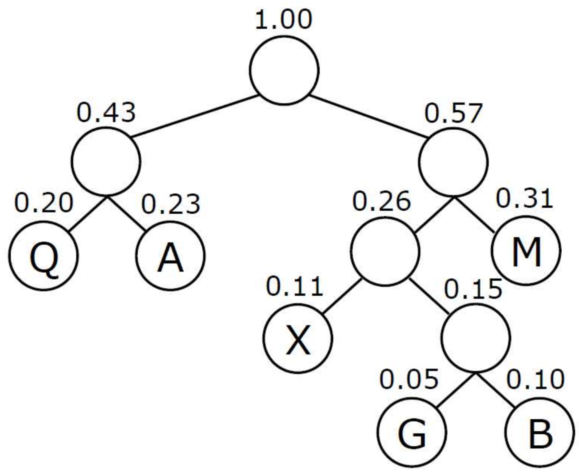
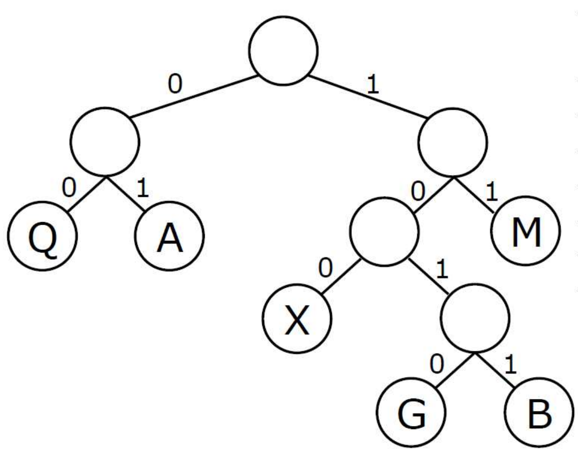
Huffman Coding: Example #3



Huffman Coding: Example #3



Letter	Code
A	01
B	1011
G	1010
M	11
Q	00
X	100

encode()/decode()

- With the dictionary for a Huffman coding assembled, it becomes very easy to encode strings:

```
huffman_codes = {
    "": '0111', 'A': '10', 'E': '1101',
    'H': '0001', 'I': '1111', 'K': '001',
    'L': '0000', 'M': '11000', 'N': '1110',
    'O': '010', 'P': '110011', 'U': '0110',
    'W': '110010'}
def encode(word, encodings):
    result = ''
    for letter in word:
        result += encodings[letter]
    return result

encode('MAUI', huffman_codes) # sample call
```


encode()/decode()

- Decoding strings is a little trickier because the dictionary's key/value pairs are reversed from what we need
  - The dictionary maps letters to codes, which is suitable for encoding
  - For decoding we need to map codes to letters
- Similar to list comprehensions, a **dictionary comprehension** lets you create a new dictionary from an existing one
- Here's the code we need. It maps a value from the **huffman_codes** dictionary back to its key:

```
reversed_codes = { huffman_codes[key]: key
    for key in huffman_codes.keys()}
```

- Would this work if values are not unique in **huffman_codes?**

encode()/decode()

- We can now write the **decode()** function, where a “reversed” dictionary is given as **decodings** :

```
def decode(encoded, decodings):
    result = ‘’
    while len(encoded) > 0:
        for i in range(1, len(encoded) + 1):
            if encoded[:i] in decodings.keys():
                result += decodings[encoded[:i]]
                encoded = encoded[i:]
                break
    return result

decode(‘110001001101111’, reversed_codes) # sample call
```

- See huffman.py for this code and examples

Questions?